World-line construction of a covariant chiral kinetic theory

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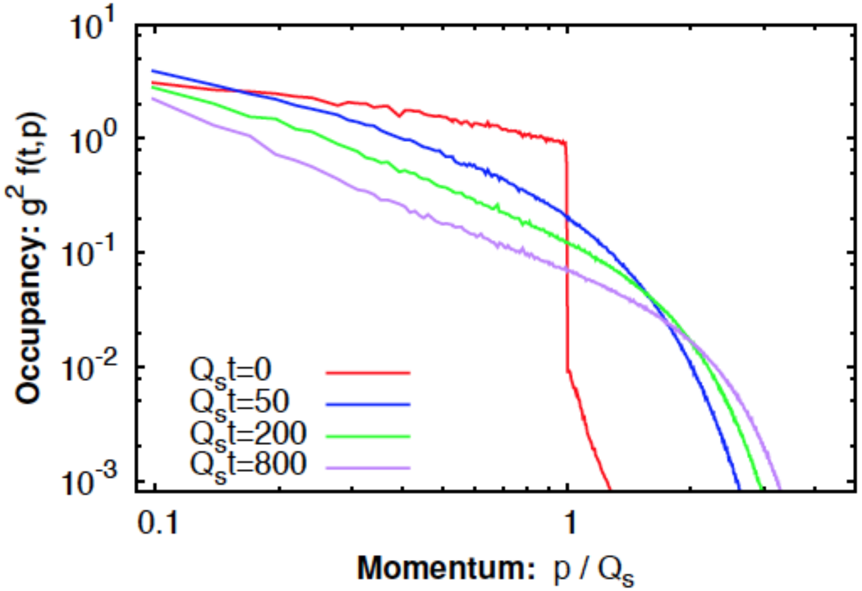
Niklas Mueller and RV, arXiv:1701.03331 and 1702.01233

RBRC workshop, February 13-15, 2017

Outline of talk

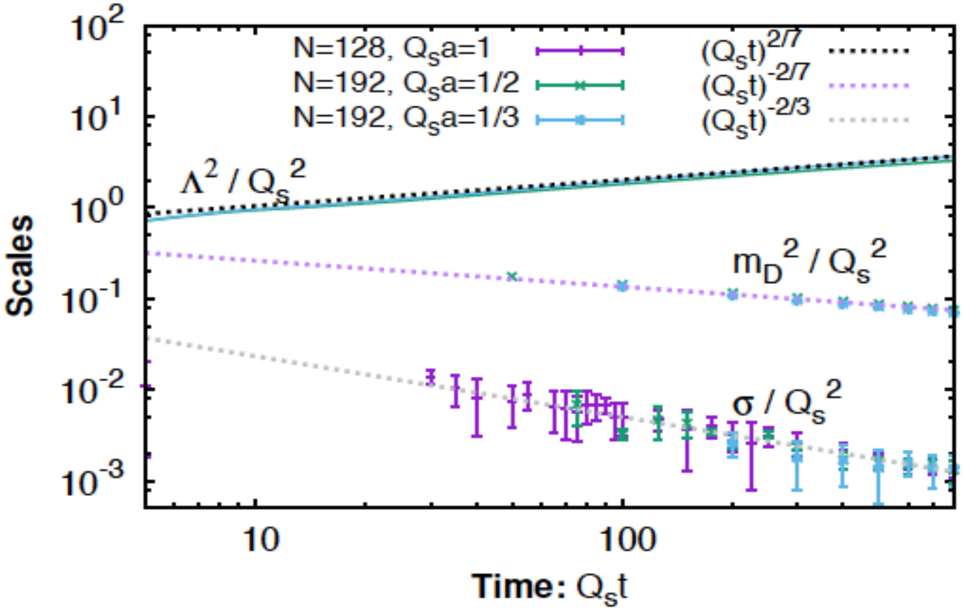
- From exploding sphalerons to chiral kinetic theory
- World-lines & internal symmetries: D'Hoker-Gagné construction of the Berezin-Marinov coherent state formalism
- Variational derivation of chiral anomaly
- Supersymmetry & Hamiltonian constraints
- Pseudo-classical Grassmanians: Bargmann-Michel-Telegdi & Wong equations
- Non-relativistic limits, Berry's phase & Fujikawa's lament
- Towards the chiral world-line Bödeker theory

Glasma: overoccupied gauge fields in a box



Thermalization extensively studied in this context employing classical-statistical simulations

Berges,Schlichting,Sexty, PRD86 (2012) 074006
Schlichting PRD86 (2012) 065008
York,Kurkela,Lu,Moore, PRD89 (2014) 074036



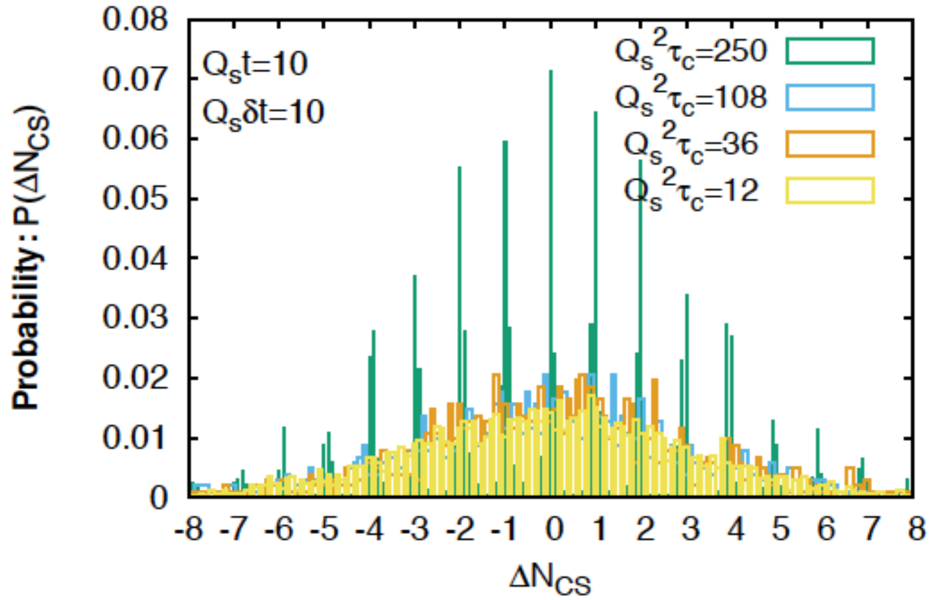
Clean separation of scales seen

Time evolution of string tension computed from spatial Wilson loops

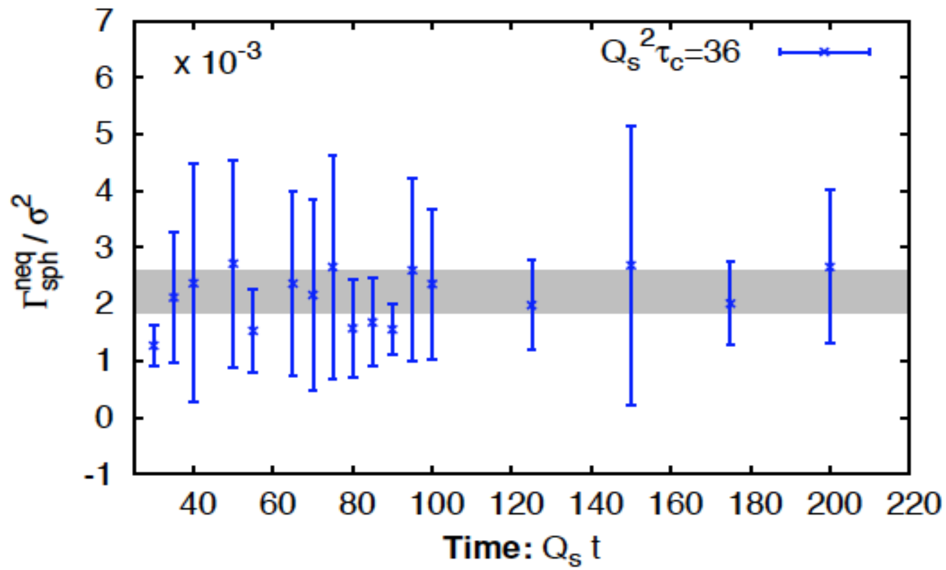
Berges,Scheffler,Sexty, PRD77 (2008) 034504
Mace,Schlichting,Venugopalan, PRD93 (2016), 074036

Topological transitions in the Glasma

Mace, Schlichting, Venugopalan, PRD93 (2016), 074036



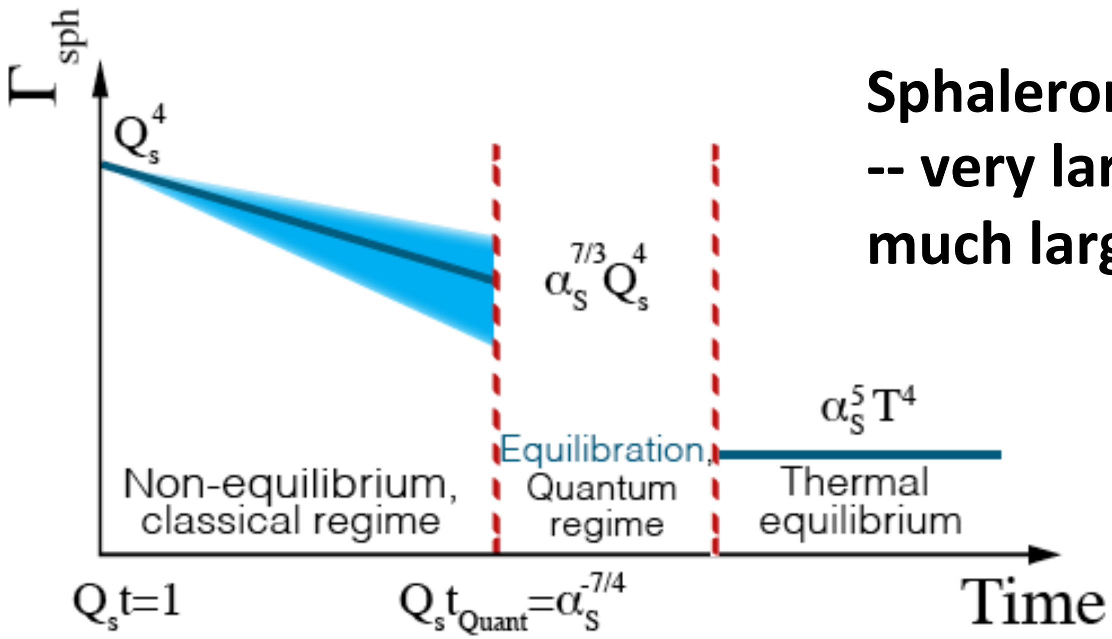
"Cooled" Glue configurations in the Glasma are topological!



Sphaleron transition rate scales with string tension squared

Exploding sphalerons

“Exploding sphalerons”: Shuryak, Zahed, PRD67 (2003) 014006



Sphaleron transition rate
-- very large in the Glasma
much larger than equilibrium rate

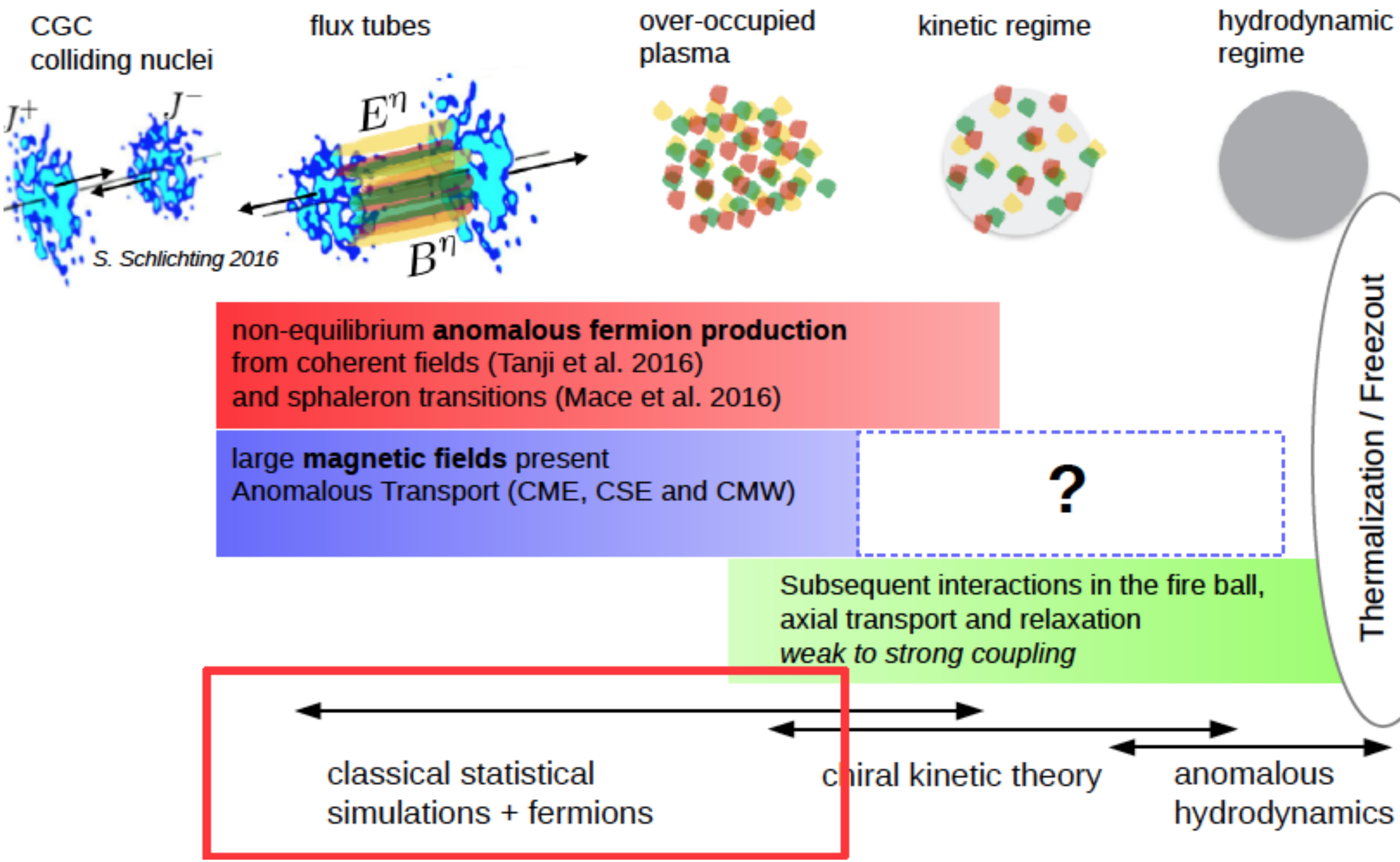
Couple sphaleron background with fermions
& external EM fields to simulate *ab initio*
the Chiral Magnetic Effect!

Mueller,Schlichting,Sharma,PRL117 (2016) 142301
Mace,Mueller,Schlichting,Sharma, 1612.02477

Talk by Schlichting

The limits of classical-statistical simulations

Slide by Niklas Mueller



Kinetic evolution of the chiral magnetic current

- Subsequent evolution of the chiral magnetic current depends on typical time scales for scattering, for sphaleron transitions, and E&M conductivity in the system

- Significant work on chiral kinetic theory

Son,Yamamoto, PRL109 (2012), 181602; PRD87 (2013) 085016
Stephanov, Yin, PRL109 (2012) 162001
Chen, Son, Stephanov, Yee, Yin, PRL 113 (2014) 182302
Chen, Son, Stephanov, PRL115 (2015) 021601
Chen,Pu,Wang,Wang, PRL110 (2013) 262301
Gao,Liang,Pu,Wang,Wang, PRL109 (2012) 232301
Stone,Dwivedi,Zhou, PRD91 (2015) 025004
Zahed, PRL109 (2012) 091603;
Basar,Kharzeev,Zahed, PRL111 (2013)161601
Stephanov,Yee,Yin, PRD91 (2015) 125014
Fukushima, PRD92 (2015) 054009
Manuel, Torres-Rincon, PRD90 (2014) 074018
Hidaka,Pu,Yang, arXiv:1612.04630

- We will discuss here a novel approach based on the world-line formalism in QCD

Niklas Mueller and RV, arXiv: 1701.03331 and 1702.01233

World-line formalism: preliminaries

Review: Corradini, Schubert, arXiv:1512.08694
Also, Strassler, NPB385 (1992) 145

Based on Schwinger's proper time trick:

$$\log(\sigma) = \int_1^\sigma \frac{dy}{y} \equiv \int_1^\sigma dy \int_0^\infty dt\, e^{-yt} = -\int_0^\infty \frac{dt}{t}\left(e^{-\sigma t} - e^{-t}\right)$$

One loop effective action of massless scalar field coupled to background Abelian field

$$\mathcal{L} = \Phi^\dagger D^2 \Phi$$

$$D_\mu = \partial_\mu - igA_\mu$$

$$\Gamma[A] = -\log\left[\det(-D^2)\right] \equiv -\mathrm{Tr}\left(\log(-D^2)\right) = \int_0^\infty \frac{dT}{T}\mathrm{Tr}\,\exp(-TD^2)$$

$$= \int_0^\infty \frac{dT}{T}\mathcal{N}\int \mathcal{D}x\,\mathcal{P}\exp\left[-\int_0^T d\tau\left(\frac{1}{2\varepsilon}\dot{x}^2 + igA[x(\tau)]\cdot\dot{x}\right)\right]$$

with $\mathcal{N} = \int \mathcal{D}p\,\exp(-\frac{1}{2}\int_0^T d\tau\epsilon\, p^2)$ ε is the Einbein: square root of 1D metric

World-line formalism: vector and axial vector fields

$$S[A,B] = \int d^4x\, \bar{\psi}\, \left(i\not{\partial} + \not{A} + \gamma_5 \not{B}\right)\, \psi$$

A is a vector gauge field and B is an auxilliary axial vector gauge field

Fermion effective action:

$$-W[A,B] = \log\, \det\, (\theta)$$ **with** $\theta = i\not{\partial} + \not{A} + \gamma_5 \not{B}$

$$W[A,B] = W_R + i\, W_I$$

Focus first on the real part:

$$W_R = -\frac{1}{8} \log\, \det \left(\tilde{\Sigma}^2\right) \equiv -\frac{1}{8} \mathrm{Tr}\, \log \left(\tilde{\Sigma}^2\right)$$

$$\Sigma^2 = (p - \mathcal{A})^2\, \mathbf{I}_8 + \frac{i}{2} \Gamma_\mu \Gamma_\nu F_{\mu\nu}[\mathcal{A}]$$

$$F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu - [\mathcal{A}_\mu, \mathcal{A}_\nu]$$

$$\mathcal{A} = \begin{pmatrix} A+B & 0 \\ 0 & A-B \end{pmatrix}$$

World-line formalism: coherent states

Doubling dimension of Dirac matrices & extension of Clifford algebra essential for coherent state spinor representation

$$\Gamma_\mu = \begin{pmatrix} 0 & \gamma_\mu \\ \gamma_\mu & 0 \end{pmatrix}, \quad \Gamma_5 = \begin{pmatrix} 0 & \gamma_5 \\ \gamma_5 & 0 \end{pmatrix}, \quad \Gamma_6 = \begin{pmatrix} 0 & i\mathbb{I}_4 \\ -i\mathbb{I}_4 & 0 \end{pmatrix}$$

D'Hoker, Gagne, hep-ph/9508131

$$\Gamma_7 = -i \prod_{A=1}^{6} \Gamma_A = \begin{pmatrix} \mathbb{I}_4 & 0 \\ 0 & -\mathbb{I}_4 \end{pmatrix} \qquad \{\Gamma_7, \Gamma_A\} = 0$$

Coherent states can be used to generate finite dimensional representations of internal symmetry groups

Berezin,Marinov, Annals Phys. 104 (1977) 336

$$a_r^\pm = \frac{1}{2}(\Gamma_r \pm i\Gamma_{r+3}), \qquad \{a_r^+, a_s^-\} = \delta_{rs}, \qquad \{a_r^+, a_s^+\} = \{a_r^-, a_s^-\} = 0,$$

$$\langle\theta|a_r^- = \langle\theta|\theta_r \qquad a_r^-|\theta\rangle = \theta_r|\theta\rangle \qquad \langle\bar\theta|a_r^+ = \langle\bar\theta|\bar\theta_r \qquad a_r^+|\bar\theta\rangle = \bar\theta_r|\bar\theta\rangle$$

$$r = 1, 2, 3$$

$$\int |\theta\rangle\langle\theta|\, d^3\theta = \int d^3\bar\theta\, |\bar\theta\rangle\langle\bar\theta| = \mathbb{I}.$$

Grassmanial path integral representation

$$W_R = -\frac{1}{8}\mathrm{Tr}\,\log\left(\tilde{\Sigma}^2\right) = \frac{1}{8}\int_0^\infty \mathrm{Tr}_{16}\,\exp\left(-\frac{\varepsilon}{2}T\tilde{\Sigma}^2\right)$$

In the fermionic coherent state Grassmanian representation, the trace can be represented as

Ohnuki, Kashiwa Prog.Theo.Phys.60 (1978)548
D'Hoker, Gagne, hep-ph/9512080

$$\mathrm{Tr}_{16}\,\exp\left(-\frac{\varepsilon}{2}T\tilde{\Sigma}^2\right) = \int d^4z\, d^3\theta\, \langle z, -\theta|\exp\left(-\frac{\varepsilon}{2}T\tilde{\Sigma}^2\right)|z,\theta\rangle$$

and rewritten as the quantum mechanical path integral…

$$\Rightarrow \frac{1}{8}\int_0^\infty \frac{dT}{T}\mathcal{N}\int_P \mathcal{D}x \int_{AP}\mathcal{D}\psi\, \mathrm{tr}_c\mathcal{P}\left(e^{-\int_0^T d\tau\, \mathcal{L}_L(\tau)} + e^{-\int_0^T d\tau\, \mathcal{L}_R(\tau)}\right)$$

with the "point particle" Lagrangian

$$\mathcal{L}_{L/R}(\tau) = \frac{\dot{x}^2}{2\mathcal{E}} + \frac{1}{2}\psi_a\dot{\psi}_a - i\dot{x}_\mu(A\pm B)_\mu + \frac{i\mathcal{E}}{2}\psi_\mu\psi_\nu F_{\mu\nu}[A\pm B]$$

Switched here from 3-D complex θ basis to that of 6-D Majorana fermions ψ_a
- simple mnemonic: $\Gamma \Rightarrow \sqrt{2}\psi$

Grassmanial path integral representation

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and rewritten as the quantum mechanical path integral…

$$\Rightarrow \quad \frac{1}{8}\int_0^\infty \frac{dT}{T}\mathcal{N}\int_P \mathcal{D}x\int_{AP}\mathcal{D}\psi\,\mathrm{tr}_c\mathcal{P}\left(e^{-\int_0^T d\tau\,\mathcal{L}_L(\tau)} + e^{-\int_0^T d\tau\,\mathcal{L}_R(\tau)}\right)$$

The vector current can be defined as (putting B^μ=0)

$$\langle j_\mu^V(y)\rangle = \frac{\delta W_R}{\delta A_\mu} = \frac{1}{8}\int_0^\infty \frac{dT}{T}\mathcal{N}\int_{\mathrm{PBC}}\mathcal{D}x\int_{APC}\mathcal{D}\psi\,j_\mu^{V,\mathrm{cl.}}\left(e^{-\int_0^T d\tau\mathcal{L}_L(\tau)} + e^{-\int_0^T d\tau\mathcal{L}_R(\tau)}\right)$$

$$j_\mu^{V,\mathrm{cl.}} = \int_0^T d\tau\,\left[\varepsilon\psi_\nu\psi_\mu\partial_\nu - \dot{x}_\mu\right]\delta^{(4)}\left(x(\tau)-y\right)$$

Can check that $\partial_\mu\, j_\mu^{V,\mathrm{cl.}} = 0$

Phase of the determinant and the chiral anomaly

- **The phase of the complex determinant is well known to be the origin of the chiral anomaly**

K. Fujikawa, PRL42 (1979)1195; PRD21 (1980) 2848

- **Its treatment in a world line path integral framework is also well known**

L. Alvarez-Gaume, E. Witten, NPB234 (1984) 269
A.M. Polyakov, *Gauge fields and strings* (1987), section 6.3

- **We will adopt a different regularization (due to D'Hoker&Gagne) and apply it to derive the quantity of interest**

Mueller, Venugopalan, 1701.03331 & 1702.01233

Phase of the determinant and the chiral anomaly

$$W_I = -\frac{1}{2}\arg\det\left[\Omega\right] \qquad \Omega = \begin{pmatrix} 0 & \theta \\ \theta & 0 \end{pmatrix}$$

with

$$\Omega = \Gamma_\mu(p_\mu - A_\mu) - i\Gamma_7\Gamma_\mu\Gamma_5\Gamma_6 B_\mu$$

Using a trick due to D'Hoker & Gagne, can be rewritten in a form *very much like that for the real part*…

$$: \frac{i\mathcal{E}}{64}\int_{-1}^{1} d\alpha \int_0^\infty dT\, \mathrm{Tr}\left\{\hat{M}e^{-\frac{\mathcal{E}}{2}T\tilde{\Sigma}^2_{(\alpha)}}\right\}$$

where the trace insertion is $\hat{M} = \Gamma_7\Lambda$

$$\Lambda = (2\Gamma_5\Gamma_6[\partial_\mu, B_\mu] + [\Gamma_\mu, \Gamma_\nu]\{\partial_\mu, B_\nu\}\Gamma_5\Gamma_6)$$

The parameter α breaks chiral symmetry explicitly
- setting it to ±1 restores it

Phase of the determinant and the chiral anomaly

The axial vector current can be expressed as

$$\langle j^5_\mu(y)\rangle = \frac{i\delta W_I}{\delta B_\mu(y)}\Big|_{B=0} = -\frac{\mathcal{E}}{32}\int_0^\infty dT\,\mathrm{Tr}\left\{\frac{\delta\hat{M}}{\delta B_\mu(y)}e^{-\frac{\mathcal{E}}{2}T\tilde{\Sigma}^2}\right\}_{B=0}$$

An advantage of the D'Hoker-Gagné construction is that the axial current has the same world-line structure as its vector counterpart

After some algebra:
i) expressing above as a path integral,
ii) separating zero & non-zero modes (the Γ_7 insertion makes PBC and Fermion zero modes feasible in the path integral construction)
iii) and fixing Fock-Schwinger gauge about the zero modes,
one can show that

See our paper 1702.01233 for details

$$\partial_\mu\langle J^5_\mu(y)\rangle = \frac{1}{8\pi^2}\mathrm{Tr}\left(\tilde{F}_{\mu\nu}F^{\mu\nu}\right)$$

which is the well known anomaly equation

Back to the real part: semi-classical world-lines

Eg., G. Dunne, C. Schubert, hep-th/0507174

Consider our simpler case of scalar particles in a background field

$$\Gamma[A] = \int_0^\infty \frac{dT}{T}\, \mathrm{e}^{-m^2 T} \int_{x(T)=x(0)} \mathcal{D}x \exp\left[-\int_0^T d\tau \left(\frac{\dot{x}^2}{4} + ieA\cdot\dot{x}\right)\right]$$

Rewrite exactly as

$$\Gamma[A] = 2\int_{x(1)=x(0)} \mathcal{D}x\, K_0\left(m\sqrt{\int_0^1 du\, \dot{x}^2}\right) \exp\left[-ie\int_0^1 du\, A\cdot\dot{x}\right]$$

$$\simeq \sqrt{\frac{2\pi}{m}} \int \mathcal{D}x \frac{1}{\left(\int_0^1 du\, \dot{x}^2\right)^{1/4}} \exp\left[-\left(m\sqrt{\int_0^1 du\, \dot{x}^2} + ie\int_0^1 du\, A\cdot\dot{x}\right)\right]$$

for $m\sqrt{\int_0^1 du\, \dot{x}^2} \gg 1$

Stationary phase "world-line instanton" of functional integral

$$m\frac{\ddot{x}_\mu}{\sqrt{\int_0^1 du\, \dot{x}^2}} = ieF_{\mu\nu}\dot{x}_\nu$$

Equation of motion of scalar particle in background Abelian field

Spinning (& colored) pseudo-classical world-lines

Brink,Di Vecchia,Howe, NPB118 (1977) 76
Balachandran,Salomonson,Skagerstam,Winnberg,PRD15 (1978) 2308
Barducci, Casalbuni, Lusanna, NPB124 (1977) 93

For a consistent treatment of the Hamiltonian dynamics, introduce Lagrange multipliers in action to impose physical constraints

$$S = \int_0^T d\tau \left\{ p_\mu \dot{x}^\mu + \frac{i}{2}\left[\psi_\mu \dot{\psi}^\mu + \psi_5 \dot{\psi}_5\right] - H \right\}$$

with $H = \frac{\varepsilon}{2}\left(\pi^2 + m^2 + i\psi^\mu F_{\mu\nu}\psi^\nu\right) + \frac{i}{2}\left(\pi_\mu \psi^\mu + m\psi_5\right)\chi$

Here ε and χ are the vierbein fields that impose the mass shell and helicity constraints of the theory

$Q=\pi_\mu \psi^\mu + m\psi_5$ is a supersymmetric charge generating an N=1 SUSY algebra

Canonical momenta:

$p^\mu \equiv \frac{\partial \mathcal{L}}{\partial \dot{x}_\mu}$ **with** $\pi^\mu \equiv p^\mu - A^\mu = m_R u^\mu - \frac{i m_R}{2z}\left(1 - \frac{m^2}{2m_R^2}\right)\left[\psi^\mu + u_\nu \psi^\nu u^\mu\right]\chi$

where u_μ is the "anomalous" velocity $m_R^2 = m^2 + i\psi^\mu F_{\mu\nu}\psi^\nu$.

Spinning (& colored) pseudo-classical world-lines

Pseudo-classical equations of motion for spinning particles in the (x,p,ψ) phase space:

$$m_R\, \ddot{x}^\mu + \frac{i}{2\, m_R} \psi^\alpha \partial^\mu F_{\alpha\beta}\, \psi^\beta + F^{\mu\nu} \dot{x}_\nu = 0$$

$$\dot{\psi}^\mu - \frac{1}{m_R} F_{\mu\nu} \psi^\nu = 0 \qquad \dot{\psi}_5 = 0$$

One can also obtain EOM for the Pauli-Lubanski vector

$$\Sigma_\mu = -\frac{i}{2} \epsilon_{\mu\nu\rho\sigma} P^\nu \psi^\rho \psi^\sigma \qquad S_\mu = \Sigma_\mu / \hbar$$

$$\dot{\Sigma}_\mu = \frac{g}{m} \left(F_{\mu\nu} \Sigma^\nu + \partial^\nu F_{\mu\nu} \Gamma_5 \right) \qquad \Gamma_5 = \psi_0 \psi_1 \psi_2 \psi_3$$

For homogeneous fields, this is the covariant form of the Bargmann-Michel-Telegdi equation

Extension of framework to colored Grassmanians gives the Wong equations for precessing color charges

Jalilian-Marian,Jeon,Venugopalan,Wirstam, PRD62 (2000) 045020

Non-relativistic limit & Berry phase

Carefully taking non-relativistic limit v/c << 1 of the world-line action,

Thomas precession term **Larmor term**

$$H \equiv mc^2 + \frac{\left(p - \frac{A}{c}\right)^2}{2m} + A^0(x) - \frac{S \cdot \left([v/c - A/(mc^2)] \times E\right)}{2mc} - \frac{B \cdot S}{m}$$

In the adiabatic limit $\frac{\mathrm{B} \cdot \mathrm{S}}{m} \approx 0$ **spin flips are suppressed and particle spin is "slaved" to its motion**

Transition amplitude from initial to final states:

$$T(p_f, p_i.+) = \langle p_f, \psi^+(p_f)| e^{-iH(t_f - t_i)} |p_i, \psi^+(p_i)\rangle$$

$$T(\mathbf{p}_f, \mathbf{p}_i, +) = \int \left(\prod_{k=1}^{N-1} d^3 p_k\right) \left(\prod_{l=1}^{N} d^3 x_l\right)$$

$$\times \prod_{j=1}^{N} \frac{1}{(2\pi)^3} e^{-i\mathbf{x}_j \cdot (\mathbf{p}_j - \mathbf{p}_{j-1}) - iH_j \Delta} \underline{\langle \psi^+(\mathbf{p}_j) | \psi^+(\mathbf{p}_{j-1})\rangle}$$

$$\exp\left(i \int dt \dot{p} \cdot \mathcal{A}(p)\right)$$

where the Berry connection is

$$\mathcal{A}(p) = -i\langle \psi^+(p) | \nabla_p | \psi^+(p)\rangle$$

Non-relativistic limit & Berry phase

Transition amplitude from initial to final states:

$$T(p_f, p_i.+) = \langle p_f, \psi^+(p_f)| e^{-iH(t_f - t_i)} |p_i, \psi^+(p_i)\rangle$$

$$T(\mathbf{p}_f, \mathbf{p}_i, +) = \int \mathcal{D}x\mathcal{D}p \, \exp\left(i \int dt \left[\dot{\mathbf{x}} \cdot \mathbf{p} - \tilde{H}\right]\right)$$

$$\tilde{H} = mc^2 + \frac{(\mathbf{p} - \mathbf{A}/c)^2}{2m} + A^0(x) - \boxed{\dot{\mathbf{p}} \cdot \mathcal{A}(\mathbf{p})}$$

Note: identical derivation if mass is replaced by large chemical potential

Note further that to recover this dynamics one has to take non-relativistic, adiabatic limits of the real part of the effective action…

The chiral anomaly in contrast arises from the imaginary phase and is independent of any kinematic limits…

Relation of Berry phase and anomaly?

Fujikawa's lament… hep-ph/0501166

The notion of Berry's phase is known to be useful in various physical contexts [17]-[18], and the topological considerations are often crucial to obtain a qualitative understanding of what is going on. Our analysis however shows that the topological interpretation of Berry's phase associated with level crossing generally fails in practical physical settings with any finite T. The notion of "approximate topology" has no rigorous meaning, and it is important to keep this approximate topological property of geometric phases associated with level crossing in mind when one applies the notion of geometric phases to concrete physical processes. This approximate topological property is in sharp contrast to the Aharonov-Bohm phase [8] which is induced by the time-independent gauge potential and topologically exact for any finite time interval T. The similarity and difference between the geometric phase and the Aharonov-Bohm phase have been recognized in the early literature [1, 8], but our second quantized formulation, in which the analysis of the geometric phase is reduced to a diagonalization of the effective Hamiltonian, allowed us to analyze the topological properties precisely in the infinitesimal neighborhood of level crossing.

and… hep-ph/0511142

What we have shown in the present paper is that this expectation is not realized, and the similarity between the two is superficial.

Towards the anomalous Bödeker theory

In hep-ph/9910299, we developed the world-line formalism that reproduced the non-Abelian Boltzmann-Langevin framework for hot QCD developed by Bödeker to describe sphaleron dynamics

Bödeker, PLB426 (1998) 351; NPB559 (2000)502
Litim,Manuel,NPB562 (1999)237

Following the construction developed here,
the framework can be extended
to construct an anomalous version of the Bödeker theory

For an alternative treatment, see
Akamatsu,Yamamoto, PRD90 (2014)125031
Akamatsu,Rothkopf,Yamamoto,JHEP1603 (2016)210

Thank you for your attention!

Berry connection and chiral kinetic theory

Son, Yamamoto,…

Canonical example: two component spinor satisfying the Weyl equation $(\boldsymbol{\sigma}\cdot\mathbf{p})u_{\mathbf{p}} = \pm|\mathbf{p}|u_{\mathbf{p}}$

has Berry connection $i\boldsymbol{\mathcal{A}}_{\mathbf{p}} \equiv u_{\mathbf{p}}^{\dagger}\boldsymbol{\nabla}_{\mathbf{p}}u_{\mathbf{p}}$

and curvature $\boldsymbol{\Omega}_{\mathbf{p}} \equiv \boldsymbol{\nabla}_{\mathbf{p}} \times \boldsymbol{\mathcal{A}}_{\mathbf{p}} = \pm\frac{\hat{\mathbf{p}}}{2|\mathbf{p}|^2}$

Fictitious magnetic field associated with a “magnetic monopole”

Son and Yamamoto consider the action

D.Xiao et al., PRL95 (2005)137204
Duval et al., PLB20 (2006) 373

$$S = \int dt[p^i\dot{x}^i + A^i(x)\dot{x}^i - \mathcal{A}^i(p)\dot{p}^i - \epsilon_{\mathbf{p}}(x) - A^0(x)]$$

with

$$\mathbf{j} = -\int \frac{d^3p}{(2\pi)^3}\left[\epsilon_{\mathbf{p}}\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} + \left(\boldsymbol{\Omega}_{\mathbf{p}}\cdot\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}}\right)\epsilon_{\mathbf{p}}\mathbf{B} + \epsilon_{\mathbf{p}}\boldsymbol{\Omega}_{\mathbf{p}} \times \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{x}}\right] + \mathbf{E}\times\boldsymbol{\sigma},$$

$$\boldsymbol{\sigma} = \int \frac{d^3p}{(2\pi)^3}\boldsymbol{\Omega}_{\mathbf{p}}n_{\mathbf{p}}$$

Relation of Berry phase to anomaly

◆ **A reading of the work of Stone et al.**

suggests that the content of the chiral kinetic equations can be obtained from the covariant BMT equation

Stone,Dwivedi,Zhou, PRD91 (2015) 025004

◆ **In our work, this arises entirely from the**

real part of the effective action…

Berry connection and chiral kinetic theory

Son, Yamamoto,…

Canonical example: two component spinor satisfying the Weyl equation $(\boldsymbol{\sigma}\cdot\mathbf{p})u_{\mathbf{p}} = \pm|\mathbf{p}|u_{\mathbf{p}}$

has Berry connection $i\boldsymbol{\mathcal{A}}_{\mathbf{p}} \equiv u_{\mathbf{p}}^{\dagger}\boldsymbol{\nabla}_{\mathbf{p}}u_{\mathbf{p}}$

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Fictitious magnetic field associated with a "magnetic monopole"

Son and Yamamoto consider the action

D.Xiao et al., PRL95 (2005)137204
Duval et al., PLB20 (2006) 373

$$S = \int dt[p^i\dot{x}^i + A^i(x)\dot{x}^i - \mathcal{A}^i(p)\dot{p}^i - \epsilon_{\mathbf{p}}(x) - A^0(x)]$$

and derive the kinetic theory relation

$$\partial_t n + \boldsymbol{\nabla}\cdot\mathbf{j} = -\int\frac{d^3p}{(2\pi)^3}\left(\boldsymbol{\Omega}_{\mathbf{p}}\cdot\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}}\right)\mathbf{E}\cdot\mathbf{B}$$